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An Instance Ontology for Structures: Their Definition, Identity, and Indiscernibility

I. Introduction

In the modern history of ideas it has been a persistent thesis that *structure*, *complexity* or *system* is ontologically and epistemologically ubiquitous and fundamental. Indeed, our common experience as well as scientific theories are of cognitive and physical domains that are each a plenum of hierarchical structures. These structures can be *static* (e.g., force vectors in equilibrium, shapes of statues) or *dynamic* (i.e., event structures, e.g., the executions of a computer programs, a tennis game); *abstract* (e.g., the Real Number System, topological spaces) or *concrete* (e.g., chairs, legal proceedings); *artificial* (e.g., machines, circuit diagrams) or *natural* (e.g., the metabolic cycles of a living bodies, quantum phenomena). It is characteristic of structures or complexes that they are wholes which are 'more than the sum of their parts', i.e., they have attributes beyond the collection of those of their constituents taken singly. That is, structures themselves have (are single relata for) further *emergent* properties and relations with definite qualitative contents or intensions that delineate what are the *sui generis* 'natures' of their subject wholes. Moreover, these emergent relations serve to interconnect their relata structures into further subsuming structures of structures, iterated up through entire hierarchies. When these hierarchies are dynamic they exhibit increasingly complex behaviors in proportion to their internal complexity. A living body, for example, is not just a 'heap' of tissue and organs, but a hierarchy of these inter-related spatio-temporally and causal/functionally in various ways, and with emergent properties and relations at each level ('ontological emergence'), e.g., metabolic functions, or consciousness at a certain level of neural complexity.[1] Likewise but in the abstract, a proposition is a complex cognitive entity with emergent properties and relations, e.g., the properties of either True or False, or logical relations with other propositions, none of which are properties or relations of sub-propositional constituents. Even simple abstract additive wholes, i.e.,

sets, mereological sums, or random 'heaps', have emergent formal properties and relations non-existent at least at the level of 'urelements' ('mereological emergence'), e.g., Element-of, Part-of, Subset-of, In-1-to-1-correspondance-with.

The explanatory power of complexity continues to be the motivation for its systematic study across disciplinary fields under the rubric of General Systems Theory.[2] More narrowly, it is a characteristic of modern science that its explicit methodology is one of generating *similarities of structure*—isomorphisms or homomorphisms—between hypothesized theoretical structures that are heavily formal and systems of observed phenomena as extended by experimental apparatus. Narrower still, there is literature in the philosophy of science arguing that the best ontological account of foundational quantum physics is structural realism, and this to the extent that at some atomic ontic level the objects-inter-related conception of structure is to be replaced by the purely relational conception of only relations-inter-related as the basic elements of physical reality.[3] Here the ultimate physical particulars and fields assay without remainder into properties and relations, an analysis that may be extended to space-time itself. The pivotal problem here, one considered by some insurmountable and thus rendering (ontological) structural realism untenable, is how there can be relational structures without supporting non-structural relata nodes or 'substances' of some sort? How this is possible is a principle contribution of the following. The analysis below responds to the fact that, despite its explanatory potential, it has remained a declared unfulfilled desideratum of General Systems Theory broadly, and of contemporary philosophy of science in particular, that there exist an adequate ontology for structures.[4] Such an ontology would provide a definitional assay of structures that accounts for their natures as 'chains' or 'lattices' of interconnected entities (which may also be structures), each structure being a whole of interconnecting relationships in specific 'mutual arrangements', and where the composing relationships are delimited by their specific contents or intensions. In the following I shall detail with precision how a realist ontology of unrepeatable unit attributes or instances provides such a definition. In contrast to nominalistic trope theory, realist instance ontology recognizes individuated relation (including property) *n*-adic instances, R^{n}_{i}, R^{n}_{j}, R^{n}_{k},... , together with sharable *n*-adic intensions (universals), R^{n}, the latter being constituent qualitative aspects numerically the same across their like in-

stances and separable only in abstraction. (The superscripts indicate the number of subject places (of not necessarily distinct subjects) per predicable union, and the subscripts have a naming function that serves to distinguish instance tokens of the same intension type.) Elsewhere I have argued in detail how the combined elements of predicable and as such unrepeatable relation instances and their non-predicable but repeatable intensions make for an ontology and implied logic (analytic/inference engine) superior to standard substance/attribute and trope ontologies and a refinement on standard predicate logics.[5] Herein I shall extend this display of power by showing precisely how the principles that yield and define instance ontology corrects traditional theses concerning plural unity and predication, and provides the otherwise elusive definition of structure or complexity, the latter a serious omission in my previous analyses. The definition and the supporting principles will yield (by then) intuitive and insightful accounts of the identity and indiscernibility of structures or complexes. Indeed, I would propose that what is the standard troublesome notion of indiscernibility is clarified only in the context of complexes as assayed below, and as evidenced by the developed class of counter-examples to the Principle of the Identity of Indiscernibles. These accounts are invisible to standard ontologies where ontic predicates are repeatable universals. In particular, if predicates are universals, i.e., sharable *types*, it would be absurd to theorize at some atomic ontic level a network of only relations-inter-related since there could be but one case for each structural form or type, viz. the type itself, a base too poor upon which to build plural reality which exhibits multiple *tokens* of identically the same types, e.g., multiple methane molecules. These problems are solved with the availability of isomorphic structures composed of corresponding instances of the same type.

In its commonly recognized form, a structure or complex is a network or mesh of variously inter-related entities, and so a definition of complexity must make use of relations understood as constituent linkings or 'mediating combinators', the 'rods', between shared object 'nodes' that together make up an inter-connected whole. Even medieval philosophers whose official doctrine was the reductive elimination of polyadic relations nevertheless recognized that it is of the nature of a relation to be a sort of 'interval' (*intervallum*)[6]—relations bridge ontological space. The assay of relations presupposed here, and, as I shall rehearse, one that implies the individuation of relations into instances, is that

each relation, insofar as it obtains among an *n*-tuple of relata (i.e., is an *ontic ('material') predicate*), is a cause of a unity of itself with and among each of its *n* subjects, and where this unity is conditioned or delimited by a specific *n*-adic content or intension, R^n (e.g., Taller-than, or Prime-Divisor-of) and its compatibility with the nature of each of the *n* subjects.[7] Exactly similar but distinct instances, R^n_i and R^n_j (e.g., $Circular_1$ and $Ciruclar_2$), are tokens of the same type R^n because intension R^n is numerically the same constituent of each of its tokens, i.e., a shared universal (*unum in multis*), a thesis that follows from the standard arguments for universals which I take to be demonstrative and shall not rehearse.[8] The point here is that relations-as-predicable, i.e., relation instances (including property instances as the limiting monadic case) are *agent ontic unifiers* that form with their relata *individuated states of plural unity*, what are the simplest and basic complexes—resultant individual *facts* or *states of affairs* (e.g., the fact that *a* is taller than *b*, or that 3 is a prime divisor of 12). In the following a colon locution will be used to distinguish a fact, i.e., $:R^n_i(a_1,a_2,..,a_n)$, from a corresponding true proposition, i.e., $R^n_i(a_1,a_2,..,a_n)$. The unity in a fact is a *plural* one—the relata are both connected via an instance of R^n and yet by the same agency held in an identity-saving distinctness from each other and the relation. In the paradigm distinct-relata case an *n*-adic relation instance predicable of its *n* relata is, indeed, analogous to a rigid connecting rod holding its subject relata via itself both linked and distinct.[9] Even in limiting cases of facts whose dyadic intension R^2 is reflexive and the subject relata are identical, e.g., the fact $a = a$, there is a plural unity of the relation instance with its distinct subject. Here the rod analogy applies by representing the two 'attaching ends' (the dyadic nature) of a reflexive instance as bent back upon and unifying itself to the same relatum.[10] This insight into the nature of (instances of) *all* relations, as each an 'intensioned linking' or 'intension delimited agent combinator', is the antidote to the sterile and/or misleading errors of founding all unity on either formal or psychological 'intensionless' concatenation (e.g., as with sets and mereological sums), or shared containment in a subject (e.g., the classic theory that attributes 'inhere in' or are 'immanent in' their substance subjects), or an identity-loosing mutual suffusion or 'blending' into a resultant homoeomerous One (e.g., Bradley's monism). The analysis also corrects the classic and influential notion of predicable 'forms' as will be outlined below.

The theses whose implications are developed herein are that all plural unity is relational and exists in its atomic form as facts, and that these facts, in turn, along with further relation instances, are the building-blocks of all other hierarchies of structures that go to make up all of reality, concrete and abstract. Closer to contemporary common experience as scientifically tutored, a traditional Aristotelian 'substance' (e.g., a man) or an artifact (e.g., a house) is now assayed as a hierarchy of structures where the 'secondary matter' consists of sub-structures and the unifying and organizing 'form' is actually multiple simultaneous relation instances existing among these structures as wholes, and where the bottom-most level of 'prime matter' is not that of incoherent bare particulars but of intensionally 'clothed' relation instances predicable among themselves. Consistent with Aristotle's conclusion in *Metaphysics* VII and VIII, it is 'forms' characterized as predicable unifers/organizers, what are in fact relation instances, that are most truly 'substance', and where, in keeping with Aristotle's other characterizations of substance, instances are also each a 'this' (particular), compose the ultimate ontic substratum, and at this atomic level are the ultimate subjects of all predication—being relata for each other.

That reality is a 'totality of facts' is a common thesis advanced by a number of philosophers, e.g., famously by Wittgenstein, Russell, and recently by D. M. Armstrong in his *A World of States of Affairs*.[11] Yet, what has remained deficient in these theories is not only a proper assay of facts but, following on this, the absence or vacuousness of proffered modes of composition among constituent facts in the formation of more complex structures, e.g., Armstrong's constructing the world from mostly mereological sums of atomic states of affairs. Central below is the demonstration of how the ontology of individuated relation instances provides the means for remediating the latter deficiency. Specifically, the argument is that facts are the simplest complexes, and all other complexes are formed recursively by further instances either sharing relata with constituent instances of given complexes, or by taking a given complex itself as a relata. By this there is an emergent transitivity of connectedness of every instances' relata with the relata of other instances so chained together. The emergent unity belongs to the entire whole but is not, or not an effect of, any single proper constituent of the whole. Here the unity of a single complex is the combined effect of a 'team'

of constituent unifiers, not a single shared constituent unifier, e.g., a form.

II. Specious Traditional Theses Regarding Unification

In most of Western philosophy the recognition of the *prima facie* interlinking or 'ontoglial' nature of relations among their own relata, as well as the possibility of an account based upon this of their mutual articulation into networks of complex wholes, were countered by the interplay of three deeply imbedded and pervasive theses. Two of the theses were erroneous in being overly restrictive, one in allowable predicate intensions and the other in the allowable number of constituent unifiers per whole. In regard to the former, the cross-subject combinatorial nature of *n*-adic relations was contradicted by the classic and false **Monadic Intension Thesis, M**:

> (**M**) *All ontic predicates are* ***monadic****, i.e., have intensions or contents that characterize their subject entities singly, e.g., Man, Horse, Circular, White.*

Relatedly and blocking a crucial insight to be exploited herein was the specious **Unity-by-the-Unit Thesis, U**:

> (**U**) ***All*** *elements making up a plural whole must share a* ***single unifier*** *as the constituent cause of their collective unity and hence of the existence of the resultant whole.*

The logical and ontological link between theses **M** and **U** was the further, and what I shall herein clarify as a potent truth, **Ontic Predication Thesis, O**:

> (**O**) *All plural unifications into wholes (that are more than the sum of their constituents) have as their immediate constituent causes the agency of intensioned combinators—ontic predicates—whereby each unifies itself with other constituents delimited by its intension, and ordered, if any, according to that intension.*

As stated, thesis **O** allows for one or more predicate unifiers per whole, it being thesis **U** of the tradition that limited these to one. That there

can be multiple predicable combinators contributing cooperatively to the unity of a single whole is a principle result below. Thesis **O** was observed in Aristotelian and scholastic ontology under the concept of predicable 'forms', the latter serving to effect all wholes of any ontological consequence, i.e., substances (e.g., a human, a horse) and artifacts (e.g., a statue, a house). The only exceptions were the loosest wholes of 'heaps' or 'groups' considered to be simply their constituents without any agent unifier (See Aquinas, *Comm. Meta.*, VII, L.17, 1673)[12]. As intimated above and argued elsewhere[13], I propose all plural wholes are 'more than the sum of their parts' in the sense of having emergent properties and relations, and because of this presupposing predicable ontic unifiers. For this reason I shall drop the phrase in parentheses in the successively more precise versions of **O** offered below. I further propose that the failure to recognize that even 'loose wholes'—heaps, sets, mereological sums, etc.—require predicable unifiers is a theoretical artifact of the distortions abetted by theses **M** and **U**.

Thesis **U** is an extensional principle requiring that a cause of unification among multiple entities be one entity connecting itself to each and so linking all and only these entities, analogous in its simplest form to, say, a thread holding together a sequence of beads, or a jar enforcing unity upon its contents (*Meta.* 1023a11-16)[14]. Paralleling **U**, thesis **M** is an intensional condition on the nexus of predication requiring that an attribute's intension specify—be appropriate for—one subject per predicable union. That is, a monadic intension specifies a predicable nexus with exactly one subject as part of its very meaning, as made explicit in grammatical predicates where the copula is added, for example, 'is a man', 'is a house', 'is circular'. The latter examples are intensions of 'pure' ontic predicates, whereas, e.g., 'is right of *a*' and 'is right of something' are intensions of 'impure' ontic predicates, where 'impure' refers to ontic predicates that are further analyzable into polyadic predicates, i.e., having more than one subject place, e.g., 'is right of'. Impure ontic predicates carry with them one or more subject places 'filled with' specific subjects or that are quantified over. Pure ontic predicates with monadic intensions could exist in a universe with a single subject entity. In contrast, a dyadic intension of a pure ontic predicate specifies a predicable nexus with exactly two subject places, as with Cause-of, Above, Square-Root-of (though for reflexive relations these subject places may

have the same entity, e.g., $a = a$). Similarly for triadic intensions, e.g., Between, and, in general, *n*-adic intensions for all *n*. In the following all reference to ontic predicates will be to pure ontic predicates. Theses **M** and **U** were perhaps considered mutually reinforcing on aesthetic grounds of symmetry or equality of proportion—that what is extensionally one is correlative with the intensionally one, and a predicable act that effects a single whole corresponds to a controlling intension specifying a single subject.

Historically, theses **M**, **U**, and **O** where utilized together most explicit and influentially in the theory of predicable 'forms' of classic Aristotelian/Scholastic hylomorphism. First and specifically in regard to **U**, Aristotle on the understanding that unity of wholeness is "in fact a sort of oneness" (*Meta*. 1023b35) further asserts that "Now most things are called one [have a unity] because they either do or have or suffer or are related to something else that is one [has a unity], but the things that are primarily called one [have the most unity] are those whose substance is one."(*Meta*. 1016b6-10; my inserts) And, "All that is is said to 'be' [and so be a unity] in virtue of something single and common"(*Meta*. 1061b13; my insert), it being also an often repeated principle in the tradition that being and unity are convertible (*Meta*. 1003b24-35, 1054a14, 1061a15). The view was reiterated later by Aquinas: "Things that are diverse do not come together in the same order [i.e., in a structured whole] unless they are ordered thereto by some one being. For many are reduced to one order by one better than many: because one is the *per se* cause of one and many are only the accidental cause of one, inasmuch as they are in some way one."(*Sum. Theo*., I, q.11, a.3; See q.103, a.3; my insert)[15] Elsewhere and assuming **U** Aquinas asserts explicitly the denial of a fundamental thesis argued herein, saying "Nor can this unity [a unity among multiple things] come from diverse ordering causes [i.e., be the collective result of multiple unifiers], because they could not possibly intend one order in so far as among themselves they are diverse."(*Sum. Contra*., I, ch.42, par.7; my inserts)[16] Later in Scotus one finds thesis **U** in the form: "Just as unity in common follows *per se* on entity in common, so too does any unity follow *per se* on some entity or other."[17] What may have motivated, or at least reinforced **U** was the causal principle that: There can be nothing in an effect that is not in the cause(s) (See Aristotle, *Meta*., 993b22-25). So, a single unification into a whole

among multiple elements cannot exist as an effect of multiple causes unless there is a single unification into a whole among these causes. But, then, the latter is just one (albeit complex) cause as is the effect. Hence, all unity among the diverse is by a single unifier. *Contra* the causal principle, the argument herein is that a complex can have emergent unity not caused by any single constituent unifier, but rather be the resultant of several.

In classic hylomorphic ontology the primary mode of unification of any ontically significant plural whole was by a *form*, substantial or accidental, ontically predicated of—being in 'act' as a unifier applied to—a subject or subjects, whether prime matter or existing substances. In *Metaphysics* VII Aristotle asserts that a predicable form answers the question "why one thing attaches to another", and it does so in the manner of a 'cause' and a 'principle', i.e., as an agent and a source from which the unity obtains, and not just as another element to be unified (*Meta.* 1041a6-41b30). Later, Aquinas is more explicit: "Each individual thing is actually a being through a form, whether in the case of actual substantial being or in the case of actual accidental being. And hence every form is an act, and as a consequence it is the reason for the unity whereby a given thing is one."(*De Spirit. Creat.*, Art. 3)[18] Here thesis **O** is explicit and thesis **U** is understood. Perhaps most explicit in assaying subject/form-predicate unification—thesis **O**—was Francisco Suarez, considered by some to be the last great scholastic philosopher and synthesizer. Suarez distinguished between a form and its union of inherence in a subject, the latter being a 'mode' of the form. The union as mode was particular and unrepeatable, yet itself neither a substance nor a quality or form of something, but rather a modification of the predicated intension. The distinction between an intension and its union of inherence was held to be post-abstraction, though with a real basis within what is an internally simple predicate (what was classified as a *distinctio rationis ratiocinatae*).[19] (Suarez's work anticipates the assay of relation instances I am proposing.) In sum then, predicable forms were considered the immediate cause of the organizing unity of themselves with their subject or subjects. For example, in a substance such as a human, the substantial form, i.e., the soul (intensionally: Humanity), was held to be, in conformity with **U** and **O**, the one and immediate *agent organizer* into a structured whole of the underlying matter. Similarly for accidental wholes such as a house or statue. Yet, in conformity with **M** and **O**, predicable forms among

plural matter and their causal role in its 'orderings' *were in every case limited to those with monadic intensions or contents*, each a species-intension rendering resulting substances one of that *kind*. The result was a 'split personality' for forms of composite wholes: specifically, the incoherence of ontic predicates each providing a structured unity among two or more subjects ('secondary matter'), yet also having an identifying intension or 'meaning' that is monadic and so specifying exactly one entity as the object of its agency. The problem is amplified when substantial form is taken to be predicated directly of prime matter (as soul was for Aquinas and Suarez). If prime matter is construed as such as an undifferentiated and amorphous single simple stratum of pure potentiality (yet as an individuator somehow numerically distinct for each distinct substance—what would have to be in the last analysis bogus bare particulars[20]), then a substantial form predicable of it must be the proximate internal cause of *both* the differentiation of multiple parts *out of* it (e.g., Socrates organs, tissues, bones, etc.), as well as the cause of the 'ordering' of the latter parts into the structured whole (e.g., into living Socrates as a complex of functioning systems). The substantial form, which is an intension or has a unique intension as a controlling aspect, is then required to be both monadic (as predicable of the bottom prime matter) and polyadic (as predicable organizer of the intermediate parts). Further and also inconsistent, the predicable act of the form as polyadic organizer here presupposes itself as *numerically the same* but prior differentiating monadic act on the internally simple prime matter. I note in this context once again the error to be corrected below that, in addition to a structuring function among differentiated parts (a role contrary to thesis **M**), a creative and nature-bestowing function effecting these parts (which is consonant with thesis **M**) is necessitated of substantial forms because they are required to be, at some foundational level at least, predicable of absolutely formless/qualityless prime matter. And, this is so because of the fallacious reasoning that in order to avoid an infinite vicious regress of analysis, what is predicable, i.e., form, requires at some base level something non-predicable, and hence formless, to be predicable of. The related and contemporary version of this fallacy is that relations (including properties) at some level require non-relational and non-predicable relata. To the contrary and answering both, we shall see that a base level of relation instances can among themselves provide both predicable unifiers and intensioned subjects.

In sum, the source of the above monadic/polyadic incoherence is the joint enforcement of two errors: the error under thesis **U** that a single complex whole of variously inter-related parts (i.e., having constituent relations of differing intensions, e.g., a human, a machine, a compound proposition) have one extensive unifier which by **O** is an ontic predicate, along with the error under thesis **M** that every agent unifier, i.e., an ontic predicate by **O**, have a monadic content that specifies one subject per emergent fact. The means for correcting **U** will be our principle effort below. Here I will indicate briefly what has been historically the insidious consequences of **M**, and what was its definitive correction by Bertrand Russell in the last century.

A principle implication of thesis **M** is the doctrine of the monadic reduction of relations, which via a number of 'emendations' has had and continues to have distorting effects, e.g., the relegation of relations to the 'supervenient' (on property reducta),[21] or the reduction of relations to associations formed by the mind (*entia rationis*) as in the Humean nonnomic analysis of causality. Reinforced is Aristotle's assessment that relations are "least of all things a kind or entity" (Aristotle, *Meta*. 1088a23). The property-reduction of relations, traceable back to Plato and Aristotle and customized variously by medieval philosophers, eliminates polyadic (historically restricted to dyadic) relations in favor of monadic properties of one or more of their relata (an *esse in* aspect) but with each of the latter having a characteristic 'being-toward' the other relata (an *esse ad* aspect).[22] Of course, to be a successful elimination of the polyadic the being-toward aspect cannot be a further albeit more subtle relation, but rather must be a kind of intensionless 'pointing'. It is but a short step to making the toward-aspect a blank association independent of the natures of the relata and freely created by the mind, a position found in the subsequent 'modern' philosophies of Spinoza, Leibniz, Hobbes, Locke, and Hume.

Abetted by these distortions, the linking 'predicable' nature of relations disappears completely in the more recent nominalistic and formal Wiener-Kuratowski strategy for the reduction of relations to certain sets of sets, where the further assumption is that sets (and similarly for mereological sums) are wholes that do not need constituent unifiers among the elements. Here relations as intension universals are given an alleged extensional reduction in terms of their relata, and the unification of ele-

ments into a set is ignored as a non-problem. The latter is related to other instances of declared non-problems found in nominalism, e.g., Ockham's assertion that "One does not have to look for a cause of individuation... Rather one has to look for the cause why it is possible for something to be common and universal."[23] In response to the Wiener-Kuratowski reduction strategy, Herbert Hochberg has shown that it is in fact unsuccessful in that it must surreptitiously appeal to ordering relations that are intensional and unreduced.[24] And, I have argued elsewhere that ignoring the necessity of internal unifiers is 'ostrich ontology' where set theory, which is a tool for formal modeling, is mis-identified with the reality modeled, and consequently, as with the Cheshire Cat in *Alice in Wonderland*, becomes analogous to a theory of grins with the ontically supporting cats (the constituent relations) abstracted away and ignored, though necessary and presupposed.[25] Plural wholes require internal causes of unity among their constituents, explicit or not.

The degeneration from intension-controlled unifiers down to blank associations or contrived formal models began with the erroneous restriction, under **M**, of the former to those with monadic intensions—the forms of classic hylomorphism. Starting only in the twentieth century has thesis **M** been widely recognized as false, and even then the ontological implications of alternative polyadic predication has received little attention. The latter accounts for the absence of overt rejections of thesis **U**. The *locus classicus* for demonstrating the error of the monadic reduction of relations, and hence of **M** that implies it, is Russell's analysis in *The Philosophy of Mathematics*.[26] The arguments turn on the non-eliminability of the ordering among relata by asymmetric and non-symmetric relations, a unique characteristic of polyadic relations, one not reducible to monadic properties singly or jointly. Elsewhere I have sought to reinforce Russell's arguments against contemporary defenders of the reductionist strategy (e.g., Keith Campbell).[27] I refer the reader there. Importantly, thesis **M** is rather to be replaced by the generalized **N-adic Intension Thesis**:

> (**N**) *An ontic predicate has an intension that specifies n subjects for a fixed n≥ 1.*

Thesis **N** now makes it possible to state more precisely the Ontic Predication Thesis, **O**, utilizing *n*-adic intensions and corresponding facts, viz.,

> (**O′**) *All plural unifications into wholes have as their immediate constituent causes the agency of intensioned combinators—ontic predicates—each having an n-adic intension* R^n *that delimits and orders (if any) other constituents into subject n-tuples,* $<a_1,a_2,\ldots ,a_n>$*, the combinator unifying these subjects into single facts,* $:R^n(a_1,a_2,\ldots ,a_n)$*, that are, or are unifying parts of, its resultant whole.*

Like **O**, left open with **O′** is the possibility of a single whole having multiple combinators and so multiple composing facts. In the next section I shall increase the warrant for **O′** and argue for a further refinement, viz., the individuation of ontic predicates. The latter will be prerequisite to correcting thesis **U**, i.e., to showing that a whole can have multiple partial unifiers whose effects 'add up' to the unity of the whole.

III. Bradley's Regress and Principles of Individuated Relations

Aristotle's argument for the unifying nature of forms (*Meta*. 1041b11-30) contains a condensed version of a historically reoccurring argument sometimes interpreted as showing the unreality of polyadic relations and now known as Bradley's Regress. Bradley himself intended correctly that the argument, if sound as he interpreted it, proves the absurd and illusory nature of all ontic predication whatsoever, monadic or polyadic.[28] The argument proceeds by observing that in the fact corresponding to the true proposition P(*a*), i.e., :P(*a*), if the ontic predicate is the intension *universal* P, e.g., if the intension Red is the ontic predicate in the fact :Red(*a*), then P (e.g., Red) and *a* are just two separate non-predicable subjects, each and in themselves making no reference to some other specific entity (neither having an 'esse ad' aspect indicative of something predicable). This is clear when fact :P(*a*) is contingent and so where P and *a* are identically the same P and *a*, respectively, that can exist when the predicable unity among them, and hence resultant fact :P(*a*), no longer exists. The unification prerequisite to contingent fact :P(*a*) requires something more than just, *per se*, intension/universal P and subject *a*. That is, the intension P as much as particular *a* are causally

inert as themselves non-unifiers and, hence, there is required some further unifier to account for the unity of the original fact :P(*a*). The non-predicable nature of intension universals, e.g., Red, Triangle, Tall, Love, is the veridical base from which Plato could launch as plausible his further and false theory of separated Forms. An intension is the same in its total being—what makes it to be what it is—in worlds with or without entities that exemplify it, and hence any principle of unification (e.g., 'participation') joining an intension and a subject entity is distinct from the intension. Now, the original fact :P(*a*) requires a unifier which, according to the regress, must now be a dyadic predicate, say the relation of Exemplification, E. The original fact then becomes the fact :E(P,*a*). But now, if it is Exemplification as a intension universal that is an element of the fact, then for the same reasons as with P, the three entities E, P, and *a* are distinct subjects none of which are connected to the other two inherently or by any nature of its own, and are in need therefore of a further unifier to account for the unity of the original fact, say, Exemplification′, E′. The original fact then becomes :E′(E,P,*a*). Clearly this is the beginning of a vicious infinite regress, where the predicate posited in the *n*-th step to account for the requisite unity is seen in the *n*+1-th step not to be capable of this role, calling for the posit of a further unifying predicate in the *n*+2-th step, and so on. Of course, the same regress results when the original fact has a polyadic predicate with *n*-adic intension R^n. Bradley concluded that "All predication, no matter what, is in the end untrue and in the end unreal... "[29] According to Bradley unity is not from ontic predication but rather from the all-encompassing One, the internally undifferentiated Absolute.

Pluralist philosophers who accept the validity of the regress argument (e.g., Ockham)[30] have no choice but to eliminate the unbridgeable (due to the regress) 'ontic distance' between each of diverse subjects and their attributes by placing the latter 'in' their subjects as container-unifiers—the classic inherence model of predication (*praedicatum insest subjecto*). The result is the forced adoption of thesis **M** and the property-reduction of relations—a *reductio*. Pluralist philosophers who reject the regress argument do so by calling into question one of its premises. These underlying assumptions are, I propose, the following three. 1) In a relational fact :$R^n(a_1,a_2,..,a_n)$ it is the relation-R^n-as-ontically-predicable-of-its-relata (what Russell termed the 'actually relating relation')[31] that is

the cause of the unity of itself with its relata and hence of the existence of the emergent fact. 2) In a relational fact $:R^n(a_1,a_2,..,a_n)$ the relation-R^n-as-ontically-predicable-of-its-relata is identical to the intension universal R^n. And 3), No intension universal is in itself ontically predicable of any subject(s). Included here are monadic properties which are the limiting case of polyadic relations. The iterated appeal to these assumptions yields Bradley's Regress. Now, historically there have been two standard responses to the regress. One is to accept propositions 2) and 3) but reject 1), holding that the cause of the unity of a relational fact is a posited implicit 'non-relational tie' or 'nexus'.[32] The trouble with this maneuver is that if the tie has a specific content or intension then it is but a further relation with the effect that the regress is only put back one step, and if, alternately, the tie has no content or intension then it becomes a 'bare linking' analogous to the specious notion of a 'bare particular' and is open to equally serious challenges (e.g., the inability to account for ordering by and direction of an *n*-adic predicate among a relata *n*-tuple).[33] The second standard response to the regress has been to retain propositions 1) and 2) but reject 3), this thought by some to be in keeping with the dominate doctrine going back to Aristotle that universals are predicable entities (*Meta*. 999b35; 1038b15). There is, however, an argument[34] that I propose is demonstrative in showing that proposition 3) is true and that it is 2) that must be rejected. In particular, the argument establishes that in a relational fact $:R^n(a_1,a_2,..,a_n)$, where with 1) *relation-R^n-as-ontically-predicable-of-its-relata* is the cause of its unity, it is the case that, *contra* 2) but implying 3), *relation-R^n-as-ontically-predicable-of-its-relata* is an unrepeatable individual and hence is not identical to the repeatable intension universal R^n. Presupposed is the non-eliminability of polyadic relations (thesis **N**), in particular contingent non-symmetric relations, and this accounts for the argument's near-invisibility to a tradition focused on monadic properties. The argument can be put succinctly as follows: Let R^2 be a contingent non-symmetric relation, e.g., Left-of, such that both facts $:R^2(a,b)$ and $:R^2(b,a)$ obtain, and $a \neq b$. The cause of the unification of fact $:R^2(a,b)$, i.e., the combinatorial act sustaining its existence as a complex whole, cannot be numerically identical to the cause in the same sense of the unity and hence existence of fact $:R^2(b,a)$. This is evident in that either fact can cease to exist while the other persists, and if it where *one and numerically the same* cause—combinatorial act—sustaining the existence of both facts, then they would have to come

into and go out of existence simultaneously, which is counter-factual. Hence, the combinatorial act sustaining fact :$R^2(a,b)$ must be unique to it, i.e., must be unrepeatable, and so particular and individual. Further and importantly, the agent cause of the unification in fact :$R^2(a,b)$ cannot be, prior to abstraction, distinct from the controlling intension R^2 in the sense that distinct implies a further implicit constituent relation between R^2 and what would be an incoherent 'bare linking'. I refer the reader elsewhere for the expanded argument.[35] This being the case, the cause of the unity of fact :$R^2(a,b)$ must be both unique to it and an internally simple combinator-under-an-intension, i.e., a relation instance R^2_i.

Generalizing then, the important ontological implication of the combinatorial nature of relations is that a *relation-R^n-as-ontically-predicable-of-its-relata* in a fact :$R^n_i(a_1,a_2,..,a_n)$ is an unrepeatable relation instance R^n_i which is a simple entity with the two abstractable aspects of repeatable intension R^n and a particularized unifying agency unique to a given *n*-tuple of subjects. The unrepeatable predicable aspect of an instance is for ontology a cogent *principium individuationis*, and cuts through the obscurities and problems associated with the alternatives of posited *haecceitas* (Scotus)[36] or bare particulars (e.g., Armstrong, Moreland)[37], the instantiation of specially endowed substance universals (e.g., Loux, Lowe)[38], or simply declaring individuation an unexplainable primitive (e.g., Ockham, Campbell)[39]. The distinction between these two real aspects of individuating combinator and its controlling intension of a nevertheless *non-complex* instance is the scholastics' *distinctio rationis ratiocinatae* (or what Scotus termed the *distinctio formalis a parte rei*).[40] The distinction applied to relation instances refines and corrects the scholastics' attribution of it to the mutual existence of form (intension + combinator) and matter (individuating subjects) in a substance, or more recently and more accurately Campbell's attribution of the distinction to the individuating and intension aspects of a (combinatorialless) trope[41], or, closer to the scholastics' use, Armstrong's use of it to characterize 'instantiation' as the "distinction without a relation" between a subject particular and its qualifying ontic predicates—the unity of a state of affairs[42]. In sum and importantly, a relation instance is as a single simple entity a 'this-such'—a 'this' because of its unrepeatable unifying agency among a specific set of relata, and a 'such' because of its repeatable intension. Alternately, an instance is both a particular and an ontic predicate. It is these facts that make possible an ontology of particulars that can be both

subjects of predication and the predicates themselves, and is the basis for how there can be structures without non-structural object nodes. This will be made clear below.

The results of the above analysis can be summarized into three of four principles that I had previously proposed as complete in properly characterizing an ontology of combinatorial predication—the realist ontology of relation instances. The first principle is the final version of the Ontic Predication Thesis[43]:

> (**O″**) *All plural unifications have as their immediate constituent causes the agency of intensioned combinators—ontic predicates—each a simple unrepeatable instance* R^n_i *with the two aspects distinguishable only in abstraction of a repeatable n-adic intension* R^n *that delimits and orders (if any) other constituents into an extension of subject n-tuples,* $\langle a_1,a_2,\ldots,a_n\rangle$*, and an unrepeatable unification on exactly one of these n-tuples effecting a single fact :*$R^n_i(a_1,a_2,\ldots,a_n)$ *that is, or is a unifying part of, the resultant whole.*

Thesis **N** is assumed in the statement of **O″**, and, as before with versions of **O** and **O′**, **O″** leaves open the possibility of multiple partial combinators (of various intensions) for a single whole, something now theoretically possible using relation instances. The second principle formalizes the unrepeatability character of ontic predicates as instances—the **Principle of Subject Uniqueness**:

> (**SU**) If $R^n_i(a_1,a_2,..,a_n)$ and $R^n_i(b_1,b_2,..,b_n)$, then $a_1 = b_1$, $a_2 = b_2$, ... , $a_n = b_n$.

This asserts that any predicate instance, R^n_i, has only one relata *n*-tuple, i.e., is not repeatable as a universal over multiple sets of subjects. A third principle is what I have called the **Principle of Relata-Linking**:

> (**RL**) No *n*-adic relation instance R^n_i exists except as ontically predicative among, and hence necessarily presupposing, some *n*-tuple of entities which as such it relates.

On the above assay it is intrinsic to the nature of relation instances that they be combinatorial among a set of relata, and hence they cannot exist separated from some such set. A fourth principle not considered above but asserting the non-redundancy of ontic predicates is what I have called the **Principle of Instance Uniqueness**:

(**IU**) If $R^n_i(a_1,a_2,..,a_n)$ and $R^n_j(a_1,a_2,..,a_n)$, then $R^n_i = R^n_j$.

The assertion under **IU** is that there can not be two distinct instances of the same intension, R^n, predicable of the same *n*-tuple of subjects, e.g., the ordered pair <3,6> will not have two instances of the relation Prime-divisor-of. The argument for **IU** is from ontic economy ('Ockham's Razor') and the fact that there is nothing to differentiate R^n_i from R^n_j here except distinct acts of predicable union, and two such unions per intension/*n*-tuple pair is redundant.

Principles **SU** and **IU** both utilize the global identity relation, =, which is easily definable in a refined predicate logic inherent in the realist instance predication specified in **O′′**. Namely,

(**Id**) Entities *a* and *b* are identical, a = b, if and only if, for every monadic property P^1 and every instance P^1_i of P^1, $P^1_i(a)$ if and only if $P^1_i(b)$.[44]

Definition **Id** asserts that entities are numerically the same if and only if they have as characterizing properties *numerically the same instances of* numerically the same intension universals. In the tradition the definition of identity without the benefit of the instance refinement (in italics) has been controversial to the extent that it was thought, rightly, not to sufficiently distinguish identity from what, given the available analytic tools, was necessarily the vague notion of 'indiscernibility'. This situation is remedied by instance ontology below.

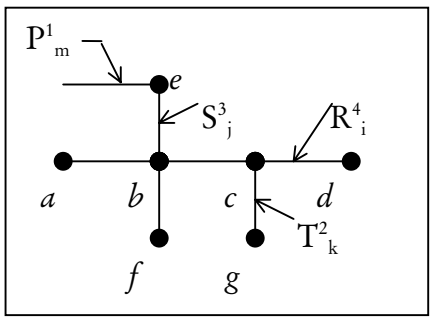
As noted, at one time I had thought principles **O′′**, **SU**, **RL**, and **IU** (or their equivalents) were sufficient to capture what is essential and potent about combinatorial predication and the resulting unit attribute ontology. I have come now to realize that omitted therein was an important principle concerning emergent unity via the proper articulation

of multiple constituent instances, what is the correction of thesis **U**, i.e., the correction of the thesis that all unity is from a single unifier. Indeed, individual facts :$R^{n}_{i}(a_1,a_2,..,a_n)$, each with their constituent trans-relata unifiers, R^{n}_{i}, do conform to thesis **U**, yet compound complexes or structures do not. The traditional error has been the false extrapolation of **U** applied to atomic complexes, i.e., facts, to its characterizing compound complexes as well. On the following, it is the ontological refinement of particularized relation instances and the possibilities for their sharing relata and having entire complexes as relata that provides an account of the emergent unity characterizing compound structures.

IV. Facts and Their Compounds

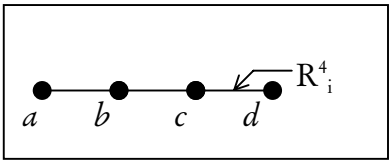
It is perspicuous on the above assay of facts that any two facts whose relation instances share one or more relata form a compound structure (though not a compound fact). The further insight to be gained is, loosely stated, that if two such pairs share a common fact, then there is a 'transitivity of unification' across all three facts forming a single more complex structure. More specifically, a trans-factual unity, what Peter Simons characterizes in topological terms as 'path connectedness'[45], emerges when pairs of complexes share relata (not necessarily the same) with mediating third complexes, analogous to the connectedness from the first to the last link in a chain without the need, to carry on the analogy, of some additional and single cable running through all the links and joining them. These claims, along with the appropriateness of the rod/node, chain, and lattice analogies used above, are made intuitive by means of spatial diagrams. These diagrams are themselves a subclass of structures whose constituent spatial relation instances are immediately observable. Consider, for example, the diagram:

Compound Complex A):

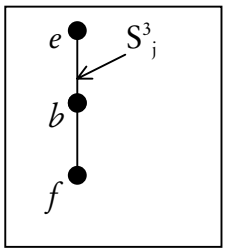


Line segments contained in the same line are to represent the single relation instance named by the terms via the arrows.[46] As a spatial complex, Complex A) displays explicitly the intra- and inter-connections among relata established via the composing facts—atomic complexes—that in less perspicuous prefix notation would be given as the conjunction of $:R^4_i(a,b,c,d)$, $:S^3_j(e,b,f)$, $:T^2_k(c,g)$, $P^1_m(e)$. Graphically, compound Complex A decomposes into the following constituent atomic complexes (facts).

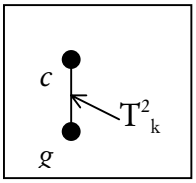
Atomic Complex B):



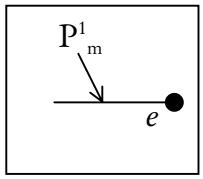
Atomic Complex C):



Atomic Complex D):

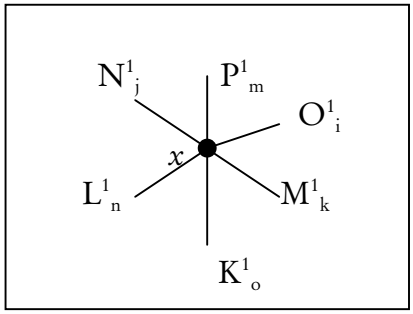


Atomic Complex E):



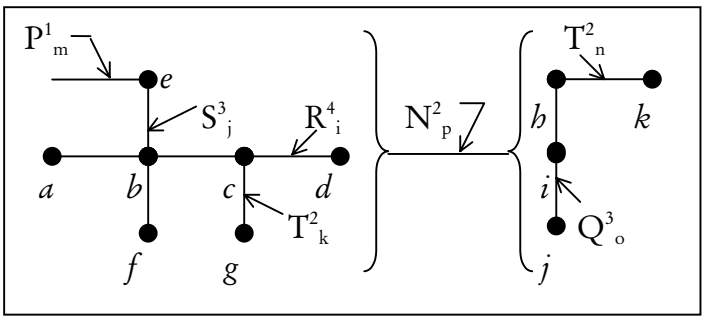
Note that in the constituent Complex E the *monadic* predicate instance P^1_m is represented by a line segment appropriately attach at one end to its single subject, *e*. Graphically then, thesis **O″** if it were restricted by the Monadic Intension Thesis, **M**, would yield a Leibnizean universe of individuals—monads—each with its halo of monadic properties, e.g., Complex F, but otherwise absolutely isolated one from the other.

Compound Complex F):



In contrast, this is not the case for the relata making up Complex A. In A, by a transitivity of connectedness via 'road and node', subject *e*, for example, is linked/unified to subject *g* by the segmented route of being a relata for instance S^3_j that also shares a different relata, *b*, with instance R^4_i, the latter in turn sharing a relata *c* with instance T^2_k that has as its other relata *g*. Here we see intuitively the 'path-connectedness' that would characterize any two constituents of a complex that is 'horizontally' composed of its relation instances, i.e., continual connectedness across sequences of facts exclusively by means of shared relata. The emergent transitivity of connectedness here is from the constituents of one fact to those of another via the facts sharing one or more relata, or from the constituents of one fact to those of another via a intermediating fact with which the two facts share one or more relata (not necessarily the same), together with the transitive nature of this connectedness relation. There is, however, in addition to horizontal composition, the important and mostly ignored 'vertical' type of composition involving relation instances having one or more relata that are themselves complexes. Consider Complex G.

Compound Complex G):



Complex G consists of an instance of the dyadic relation N^2 linking Complex A in its entirety and as a single entity as a left relatum with a different complex, say H, taken in its entirety and as a single entity as a right relatum. Analogically, Complex A could represent the structure of a tea cup, H the structure of a saucer, and relation N^2 the Sits-up relation, or, more directly, A and H could represent molecular structures and N^2_p an instance of the Has-as-a-catalyst relation. Importantly, even though between the constituents of A and H there is clearly no 'path-connectedness' by any continual chain of relata sharing 'path segments' (that are all constituents of G), nevertheless it is intuitive that relatum *e*, say, is linked to relatum *j* via a 'once-removed' next-level relation instan-

ce N^2_p. Instead of being path-connected, we might say that *e* and *j* are 'cross-level-connected'. This is so by what I shall call the 'vertical connectedness' *e* and *j* each have with N^2_p—*e*, say, is not a relatum for N^2_p but is a constituent of an encompassing Complex A that is a relatum for N^2_p , *e* being presupposed by but 'once-removed' from the combinatorial agency of N^2_p. So *e*'s connectedness to N^2_p (and to any entity N^2_p is connected to) is inherited via the mediating Complex A. Similarly for *j*. Now, it is easy to imagine this type of horizontal combination repeated on Complex G itself—G being a single relatum for other *n*-adic relations, some emergent at this level, and where this vertical structuring can be iterated into hierarchies of increasingly compound complexes. In this hierarchy constituents of the lowest level complexes would be linked or connected to any constituents of complexes at any higher level via a transitivity of unity across chains of vertically and/or horizontally composing relation instances.

Complex G exhibits what are the three and only three types of plural unification, all via relation instances: unity among relata, among relation instances, and among complexes. We can now generalize from Complex G to a full definition of Complexity (or Structure) given recursively in the following axiom for all plural unifications. The axiom is the awaited correction of the Unity-by-the-Unit Thesis, **U**, i.e., corrects the thesis that every plural unity requires a single constituent unifier among all other constituents. We have the **Unity-by-Instances Thesis, I** :

(**I**) All plural unity—complexity or structure—is by the following:

a) A relation instance R^n_i predicable of an *n*-tuple of relata, $<a_1,a_2,..,a_n>$,
is the cause of an individual plural whole, i.e., a fact $:R^n_i(a_1,a_2,..,a_n)$,
having R^n_i, a_1, a_2, .., a_n, as its only constituents.

b) If R^n_i is a constituent of a plural whole *x* and S^n_j is a constituent of
a plural whole *y*, and R^n_i and S^n_j, share one or more relata, then there

is an individual plural whole z that has as constituents all and only the combined constituents of x and y (horizontal composition).

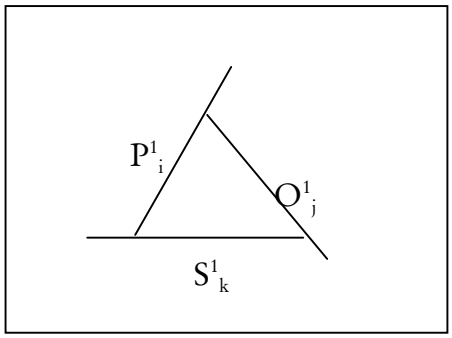
c) For any fact $:R^n_i(a_1,a_2,..,a_n)$, if for $1 \leq j \leq n$, a_j is a plural whole, then there exists an individual plural whole whose constituents are all and only the constituents of the fact and constituents of a_j (vertical composition)

It is now a simple matter to give an identity criterion specific to complexes (utilizing the global identity relation defined in **Id**), a criterion that, importantly, involves only the *internal* and so relevant nature-bestowing components of a complex *qua* complex. It is intuitive that complexes with numerically the same relations (instances) each having corresponding relata that are numerically the same are themselves numerically the same—identical. Under standard ontology where predicates are treated as universals the latter condition would not be guaranteed. However, under the Principle of Subject Uniqueness, **SU**, predicates are particularized to specific relata n-tuples, and so instances that are numerically the same have corresponding relata that are numerically the same. Hence, the **Identity Criterion for Complexes**, **ID**, is simply:

(**ID**) For complexes x and y, $x = y$ if and only if, for every intension R^n and every instance R^n_i of R^n, R^n_i is a constituent of x if and only if R^n_i is a constituent of y.

Shortly and as promised we shall consider how the ontology of relation instances clarifies the concept of indiscernibility. As a preliminary I shall make-good on another important claim repeated above, viz., that at some lowest ontic level it is possible to have only predicable entities (relation instances), i.e., predicates that have as their subjects only further predicable entities. At this base level there are no non-predicable 'substances', but only individual complexes exclusively composed of instance predicates. Consider as a perspicuous example of such a lowest level whole Complex I which is composed of a chain of monadic instances circularly predicated of one another.

Compound Complex I):



In prefix notation, the three predications here are given in the facts $:P^1_i(S^1_k)$, $:S^1_k(O^1_j)$, and $:O^1_j(P^1_i)$. Clearly, Complex I is a plural whole composed of only predicable individuals—monadic instances O^1_j, P^1_i, and S^1_k—with *no non-predicate subjects*. The same situation is possible for combinations of any *n*-adic instance predicates as long as each of their *n* subjects is itself an instance predicate. Depicting graphically such complexes would be increasingly difficult, requiring the use of curved lines for instances, and best done in three-dimensions. We need not pursue that here. Once we have such basic complexes it is easy to conceive of these wholes extended iteratively both horizontally and vertically up through hierarchies of increasingly complex structures. For example, 'instance-only' structures such as Complex I could be the relata *a*, *b*, *c*, etc., in Complex G above, as such vertically connected to G's composing instances. In sum and with an importance for ontology that cannot be overstated, what these examples substantiate is the possibility that all complex individuals whatsoever can be built up exclusively from, and by means of, predicable combinators from the single category of relation instances. Or in the reverse direction of analysis, not all predication necessarily presupposes non-predicable subjects ('substances' or their reducta of bare particulars), but that there can be an atomic ontic level of mutually-sufficing predicable individuals from which all other individuals (compound complexes) are derived—in instance ontology there need not be the regress to absurdity of 'turtles all the way down'. This insight does not contradict the maxim that 'There are no relations (and hence structure) without relata', but corrects the prevailing preconception that a system of relations always presupposes a base level of relata nodes that are 'more substantial' non-relational, non-predicable entities.

V. The Indiscernibility but Non-Identity of Certain Structures

We are now in a position to clarify the concept of indiscernibility, and indeed to illustrate how there can be numerically distinct but indiscernible entities based upon a properly understood 'internalist' criterion of indiscernibility that compares what is their total composing predicable constituents, i.e., relation instances. Hence, with this we have in a perspicuous manner the contingent falsity of the Principle of the Identity of Indiscernibles. In addition to its importance to ontology generally, the topic of indiscernibility is presently of acute interest in the philosophy of science and concerns the 'loss of identity' or 'metaphysical underdetermination' of sub-atomic entities under quantum mechanics. Operative here is the 'Indistinguishability Postulate' of quantum statistics which asserts that permutations of particles of the same kind are not observable (in making no difference in the probabilities of measurement outcomes).[47] The underdetermination debate has to do with whether quantum entities are 'individuals' (what are often described in this context as sets of intrinsic properties (e.g., rest-mass, charge, spin, etc.) individuated each by a non-qualitative something), or 'non-individuals' (i.e., entities that are in their very identity and nature somehow vague, however this can be understood ontically as opposed to simply modeled formally, e.g., with 'quasi-sets'[48]). In response, (ontological) structural realism has been put forth as, among other things, an alternative that explains the individual/non-individual dichotomy as two ways of conceiving the same structural reality.[49] Here there is a reconceptualization of electrons, elementary particles, etc., in structural instead of individualistic terms, one where the usual relationship of ontic priority between objects and encompassing structures—systems of relations among and together with these presupposed relata objects—is inverted at a foundational level so as to exist between systems exclusively of relations and resultant objects built up from them. We have just seen—with analogs of Complex I—how the latter is possible without a vicious regress of presupposed relata objects. Building upon this analysis we shall see now how a realist instance ontology can provide distinct complex wholes—'objects'—that conform to the Indistinguishability Postulate and yet are each properly characterized as an *individual* in a sense that corrects the distorting bundle-of-universals-plus-individuator or alternate trope-bundle conceptions, and, moreover, provides a precise criterion based upon internal constituents whereby these distinct individuals are indiscernible. Whe-

ther in fact quantum entities can be assayed as such 'objects' is, of course, for the structuralist program in the philosophy of physics to determine. The analysis here is offered as providing the detailed ontological underpinning for such a program.

On the above all entities, with the exception of founding and component relation instances, are complexes in the precise way given, a detailed internal analysis of entities invisible to traditional substance/attribute ontology. In the latter, any internal analysis of a subject substance *a* consisted of either a single form predicable of the other parts and/or prime matter of *a*, or the monadic properties predicable of *a* bundled together, with or without an additional individuator, to constitute *a*. It was, however, in the cruder context of substance/attribute ontology that our intuitive concept of indiscernibility as *qualitative sameness* was first standardized as the formal criterion $(F)[F(x) \equiv F(y)]$, and in which continues the controversy over the concomitant Principle of the Identity of Indiscernibles, i.e., that indiscernible entities so defined are identical, or symbolically, $(F)[F(x) \equiv F(y)] \supset x = y$. The lack of progress in the latter controversy is, I propose, symptomatic of an error in the standard formal criterion for indiscernibility, and thus in the motivating ontology that can analyze the internal nature of entities only by making essential use of derivative externally predicated attributes of them: their species 'forms' or all their monadic attributes. The intuitive indiscernibility concept of 'qualitative sameness' is synonymous with 'same in every way that is identically repeatable'. If, as was the case in much of the tradition, an ontology recognizes only monadic intensions as numerically repeatable qualitatively characterizing entities, and holds that an entity characterizes a subject by being ontically predicated of it, then indiscernibility between any *x* and *y* does indeed reduce to $(F)[F(x) \equiv F(y)]$. But this makes indiscernibility dependent upon *external* predicates posterior to the subject entities compared, and so confuses the debate over the identity of indiscernibles with tangential and inconclusive arguments why *prima facie* irrelevant external properties like 'is identical to *a*', 'is different from *b*', 'is two units from *a*' are (or are not) indeed irrelevant to indiscernibility.[50] The intuition that indiscernibility is a matter of the internal constitution of entities is what motivates in this context the attempts to distinguish 'intrinsic' for 'extrinsic' and 'pure' from 'impure' properties, and to make indiscernibility in its strongest form turn upon

'pure intrinsic' properties.[51] The same intuition is found expressed in Leibniz's formulation of the Principle of the Identity of Indiscernibles: "There are never two beings in nature which are perfectly alike and in which it is impossible to find a difference that is *internal* or founded on an *intrinsic denomination*."(my italics)[52] For Leibniz the properties of an entity *a* are both predicable of *a* and together compose *a*'s 'complete concept'. By the Identity of Indiscernibles the complete concept of *a* is unique to it since no two individuals can have the same bundle of characterizing properties. Moreover and conversely, the Identity of Indiscernibles follows from the assumption that the universal properties predicable of an entity *a* are all and only the constituents of *a*, together with an intuitive thesis known as the Principle of Constituent Identity: Complete identity in corresponding constituents of *a* and *b* entails numerical identity of *a* and *b*.[53] Of course, the problem here is that an unrepeatable particular cannot be identical to all and only its repeatable properties bundled together since the bundle itself is thus repeatable. What is missing, and telling of the error of the whole analysis, is an individuator but one that, by the same analysis, would in the end have to be a bare particular. In contrast, the other premise—the Constituent Identity principle—is intuitive, and, indeed, when formalized and applied to complexes is our above concluded Identity Criterion for Complexes, **ID**. What would be the corresponding and equally apparent principle for indiscernibility is that: Complete indiscernibility between both corresponding structures (isomorphism) and the corresponding entities structured that jointly make up each of *a* and *b* entails the indiscernibility of *a* and *b*. What is required, then, is that we render precise these pre-critical intuitions concerning how indiscernibility is a matter of the *internal* nature or constitution of entities, what in the primary sense makes things to be what they are and not something else, and what is presupposed by properties and relations that have these entities as relata (e.g., spatial relations). This is a project now possible in the refined context of structure theory built from instance ontology.

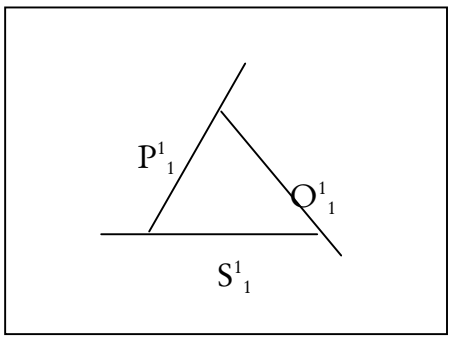
Founding the project of an internal criterion for indiscernibility is the fact that in the realist ontology of relation instances there are atomic entities—individuated ontic predicates—that are absolutely qualitatively the same and yet distinct. Specifically, two distinct instances R^{n}_{i} and R^{n}_{j} (e.g., $Square^{1}_{1}$ and $Square^{1}_{2}$) of the same type R^{n} (e.g., $Square^{1}$) are qualitatively indistinguishable in the precise sense that each shares as their to-

tal qualitative content numerically the same intension R^n, and, importantly, where as such intension R^n is not an ontic predicate of (not external to) the instances it characterizes. What renders R^n_i and R^n_j distinct is not a difference in intension or qualitative content, but rather distinct combinatorial aspects, i.e., distinct predicable 'acts', that are 'formal' in adding nothing to the concomitant intensions of their respective instances. The combinatorial aspect of an instance R^n_i is not a further intension in addition to R^n, but a unifying functionality of R^n_i specified in its range and ordering (if any) by the instance's other aspect and sole intension R^n. It is this pivotal insight that cuts through the unhappy traditional alternatives of entities having to differ either only numerically (*solo numero*), i.e., without any internal difference whatsoever, or in some intensional aspect, or by some posited but unanalyzable constituent individuator (e.g., *haecceitas*, bare particulars). Relation instances can differ by their non-qualitative but combinatorial aspects. Hence, instances R^n_i and R^n_j of the type R^n are intensionally identical but numerically distinct, and so straightforwardly *indiscernible but not identical*. So, at this point we have a precise notion of indiscernibility based upon internal aspects of entities (i.e., identity of constituent non-predicable intensions) and a refutation of the Principle of the Identity of Indiscernibles, but only, though crucially, for the limiting case of relation instances. Yet, instances R^n_i exist only as constituents of facts, $:R^n_i(a_1,a_2,..,a_n)$, and all the plural entities making up reality are facts or their compounds. What is required is extending this analysis to structured entities built up from instances.

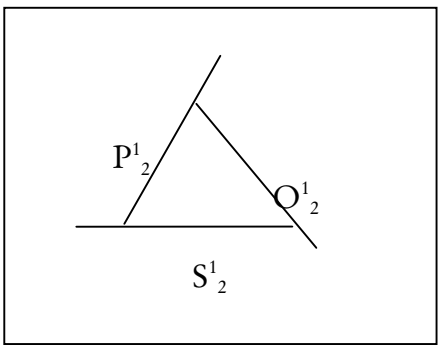
The question is: How is it that *complex entities* built up from relation instances satisfy the intuitive indiscernibility criterion of 'same in every way that is identically repeatable'? As a first approximation and as noted above, it is apparent that indiscernibility so conceived would mean for structured entities *exact ontic congruence*—an exact matching of constituents that preserves both all formal structure to the last detail and all qualitative aspects of *all* the constituents (viz., the intensions of the respective linking instances and the qualitative content of the respective relata linked). For what is repeatable is both structural form and the qualitative aspects of the entities making up the structure. To reinforce this, consider first the lowest level of complexity, i.e., individual facts. It is evident for facts $:R^n_i(a_1,a_2,..,a_n)$ and $:R^n_j(b_1,b_2,..,b_n)$, whose instances, R^n_i and R^n_j, have the same intension R^n, that they are indiscernible if and on-

ly if a_k is indiscernible from b_k, for all k, $1 \leq k \leq n$. That is, because the facts have the same predicate intension *and* because of the isomorphism that exists between the facts' relata n-tuples $\langle a_1,a_2,..,a_n\rangle$ and $\langle b_1,b_2,..,b_n\rangle$, due to their being identically ordered by this same intension R^n, the only thing that could qualitatively distinguish these facts internally is some qualitative difference in respective relata. Without this difference, i.e., with indiscernible respective relata, subsuming complexes $:R^{n_i}(a_1,a_2,..,a_n)$ and $:R^{n_j}(b_1,b_2,..,b_n)$ are themselves indiscernible, and this is possible in a non-circular way when the correlative relata are indiscernible in the prior manner of relation instances. What is intended here can be seen in the example complexes J and K, which are cases of complex I above.

Compound Complex J):

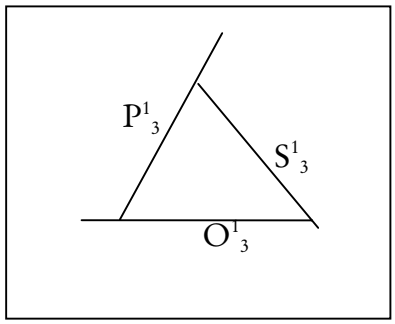


Compound Complex K):



Complexes J and K are isomorphic in form where each constituent instance corresponds to one of the same intension—P^1_1 and P^1_2 having intension P^1, O^1_1 and O^1_2 having intension O^1, S^1_1 and S^1_2 having intension S^1—and more importantly, indeed crucial to any satisfactory analysis of possible indiscernibility for complexes, the corresponding relata for the corresponding instances are themselves instances of the same intension and so, on the above analysis, are guaranteed to be internally indiscernible. Stated otherwise, complexes J and K are indiscernible because they each decompose without remainder into relation instances such that there is a one-to-one correspondence between them that preserves the relative ordering or structure within each complex, i.e., an isomorphism between J and K, and where the corresponding subject or relata 'nodes' are instances of the same intension or kind. The concept can be clarified by comparing complexes J and K with the following complex L.

Compound Complex L):



There is a one-to-one correspondence between the composing instances of L and those of J and K, but the latter complexes are discernible for L on the basis of the different relative ordering (non-isomorphism) of their instances, e.g., :$P^1_1(S^1_1)$ in Complex J but :$P^1_3(O^1_3)$ in Complex L.

We can, of course, have the same indiscernibility on complexes of atomic instances of any *n*-adicity, not just the monadic as in complexes J and K, if the instances composing each complex have other instances of that complex as relata and this mutual predication corresponds isomorphically across the two complexes. Moreover, it is possible to maintain indiscernibility across horizontal compositionality, as long as corresponding substructures are isomorphic and decompose into exactly matching instances of the same intensions, and these instances' corresponding relata are isomorphic and decompose likewise, and so on, until at an atomic level there are only corresponding isomorphic and vertically composed complexes, e.g., such as J and K. Consider, for example, two cases of complex G that differ only by subscripts on their respective predicate terms and where corresponding relata, say *a* and *a′*, are indiscernible complexes like J and K.

We can now formulate our post-critical notion of indiscernibility for complexes in its complete generality, i.e., for compounds composed both horizontally and vertically. Namely, complexes are indiscernible if and only if they are isomorphic (structurally congruent), which includes all corresponding substructures taken as single relata, and that the atomic composing instances of every set of corresponding substructures are identical in intension. This educated intuition of indiscernibility for

complexes is then made precise with the following formal recursive definition **IND**. The recursive form of the definition guarantees the requisite isomorphisms at each structural level.

(**IND**) Entities x and y are indiscernible if and only if

1) $x = R^n_i$ and $y = R^n_j$, where R^n_i and R^n_j are instances of the same intension R^n.
2) $x = :R^n_i(a_1,a_2,..,a_n)$ and $y = :R^n_j(b_1,b_2,..,b_n)$ and a_k and b_k are indiscernible for $1 \leq k \leq n$.
3) x and y are complexes such that there is a one-to-one correspondence f between their constituent facts where $f(:R^n_i(a_1,a_2,..,a_n)) = :R^n_j(b_1,b_2,..,b_n)$ and where $:R^n_i(a_1,a_2,..,a_n)$ and $:R^n_j(b_1,b_2,..,b_n)$ are indiscernible.[54]

Note that as it should be, under **IND** a complex x is indiscernible from itself since clauses 1)—3) allow for the case of $R^n_i = R^n_j$. But, of course, the whole point of the previous analysis was to show that, given a realist instance ontology, it is possible for there to be instances R^n_i and R^n_j such that $R^n_i \neq R^n_j$ and that they and their corresponding facts satisfy clauses 1)—3). That is, there can be iterated hierarchies of complexes, horizontally and/or vertically composed, and conforming to **IND** that are numerically distinct but indiscernible according to their internal form or structure and respective qualitative/intensional contents of the constituents. Hence the contingent falsity of the Principle of the Identity of Indiscernibles.

With this we now have an account of how distinct entities can satisfy the Indistinguishability Postulate of quantum physics. If fundamental physical entities, particles or fields, are 'completely relational' in the sense made precise herein and indiscernible according to **IND**, then with the permutation of such complexes within larger subsuming structures, e.g., those including the added system of relations and entities introduced by instrumentation, what will change in the total subsuming structures before and after the permutations is *only* the individuality (i.e., the combinatorial aspects) of the composing instances, not the intensions of the instances or the isomorphisms between the structures. Hence, any instance of a property emergent on such an entity+instrumentation

complex and representing a measurement on the entity will differ only individually, i.e., as a different instance, from the instance of that property that will emerge on the indiscernible complex that results with the permutation of an indiscernible entity. Indiscernible measurement structures on indiscernible but distinct complexes entities effects indiscernible but numerically distinct measurement results. On the above, however, this does not imply that indiscernible quantum entities are not-individuals or in their very natures vague entities.

VI. Conclusion

It is a theme of contemporary ontology and foundational quantum physics that reality is inherently relational. We have seen how a realist instance ontology of individuated *n*-adic ontic predicates (instances) and their sharable non-predicable intensions can correct traditional theses regarding unification and account for such a world of pure and qualitatively multifarious structure, and this all the way down to an atomic ontic level of only inter-predicable relation instances. No ultimate non-predicable subjects (substances, prime matter, bare particulars, non-predicable relata) need be posited. The combinatorial—agent unifier—aspect of an ontic predicate provides ontology with a non-posited *principium individuationis*, and it together with its formally distinguishable concomitant intension aspect, makes for a category of *intensionally controlled* (as to their compatible subjects) *individuated unifiers*, what are the atomic ontic links (and ultimately what is linked as well) that make up the hierarchical lattice this is reality. It is these property and relation instances that make possible a precise recursive definition of structure or complexity, as well as provide the basis, missing in the tradition, for an internalist or constituent criterion for structural identity and indiscernibility, the latter definable recursively. The resulting analytic precision makes it possible to display perspicuously how there can be indiscernible but non-identical entities (structures).

Notes:

1. For arguments for ontological emergence, especially from quantum mechanics, and its distinction from epistemological emergence see M. Silberstein and J. McGeever, 'The Search for Ontological Emergence', *The Philosophical Quarterly* 49 (1999), pp. 182-200.

2. E.g., and classically, L. von Bertalanffy, *General Systems Theory* (New York: George Graziller, 1969); Ervin Laszlo, *Introduction to Systems Philosophy* (New York: Gordon & Breach, 1972). There is an extensive literature on systems theory, and an active ongoing interest as evident from an internet search.

3. See James Ladyman, 'What is Structural Realism?', *Studies in History and Philosophy of Science* 29 (1998), pp. 409-24. Steven French and James Ladyman, 'Remodelling Structural Realism: Quantum Physics and the Metaphysics of Structure', forthcoming in *Synthese*. Including an analysis of the historical structuralism of Cassirer and Eddington is Steven French's 'Symmetry, Structure, and the Constitution of Objects', in the *PhilSci Archives*, Center for the Philosophy of Science, University of Pittsburgh at http://philsci-archive.pitt.edu/. For a more critical view of structural realism see in the same *PhilSci Archives* Anjan Chakravartty, 'The Structuralist Conception of Objects'. For a trope analysis of the 'relations-inter-related' conception of foundational physics see Andrew Wayne, 'A Trope Ontology for Classical and Quantum Field Theory', forthcoming in a volume edited by W. Myrvold in the *University of Western Ontario Series in Philosophy of Science* (Kluwer). Also advocating trope theory in this regard is Peter Simons, 'Particulars in Particular Clothing: Three Trope Theories of Substance', *Philosophy and Phenomenological Research* LIV (1994), pp. 553-75. Building upon the analysis given herein, my claim is that, in regard to the current debate over 'ontological structuralism' in the philosophy of science, a realist ontology of unit attributes is superior in explanatory power to nominalistic trope theory.

4. That we have not had an adequate concept of complexity or structure is a complaint of ontologically sensitive system theorists. The reason was identified as far back as by J. H. Marchal in 'On the Concept of a System', *Philosophy of Science* 42 (1975), pp. 448-68, viz., "A general account of when a relation or set of relations *holds among* the members of a set is still needed." Such an account is provided herein, it being only possible within an (realist) instance predicate ontology.

5. D. W. Mertz, *Moderate Realism and its Logic* (New Haven: Yale University Press, 1996). The ontology is described succinctly in the more recent 'Combinatorial Predication and the Ontology of Unit Attributes', *The Modern Schoolman* LXXIX (2002), pp. 163-97, and 'Individuation and Instance Ontology', *Australasian Journal of Philosophy* 79 (2001), pp. 45-61. The particularized predicate logic (PPL) inherent in the instance ontology and given initially in *Moderate Realism* is perfected in 'The Logic of Instance Ontology', *Journal of Philosophical Logic* 28 (1999), pp. 81-111.

6. See Jeffrey Brower, 'Medieval Theories of Relations', *The Stanford Encyclopedia of Philosophy* (Summer 2001 Edition), Edward N. Zalta (ed.), URL = http://plato.stanford. edu/archives/sum2001/entries/relations-medieval/

7. See my 'Combinatorial Predication' and 'Individuation and Instance Ontology'.

8. E.g., as found in D. M. Armstrong, *Nominalism & Realism: Universals & Scientific Realism*, Vol. 1 (Cambridge: Cambridge University Press, 1978) , and Reinhardt Grossmann, *The Categorical Structure of the World* (Bloomington, IN: Indiana University Press, 1983).

9. Even F. H. Bradley, one of the best-known modern antagonist of the reality of relations, noted that it is of the nature of a relation (at least if it is not reflexive) to be both a 'between' and a 'together, by which he meant that a predicable relation has a mediating role of holding its relata both apart and distinct as well as unified or joined, analogous to a rigid connecting rod between thus linked but separate subject nodes. See Bradley's 'Relations' in *Collected Essays*, Vol. 2 (Westport, CT: Greenwood Press, 1970), pp. 634ff.

10. Such diagrams are found, for example, in Rudolf Carnap's *Introduction to Symbolic Logic* (New York: Dover, 1958), pp. 118.

11. Ludwig Wittgenstein, *Tractatus Logico-Philosophicus* (London: Routledge & Kegan Paul, 1961), p. 7, Prop. 1.1; p. 13, Prop. 2.05. Bertrand Russell's views on facts are distributed throughout his works, but is succinctly put in his summary *My Philosophical Development* (London: Allen & Unwin, 1959), pp. 112-13. D. M. Armstrong, *A World of States of Affairs* (Cambridge: Cambridge University Press, 1997). For an alternate analysis of concrete reality as purely structural, one built up from a single dyadic symmetric relation, see Randall Dipert, 'The Mathematical Structure of the World: The World as Graph', *The Journal of Philosophy* XCIV (1997), pp. 329-58. This sort of reduction of reality to a single 'kind' of relation smacks of an a priori formal modeling, whereas the analysis herein allows reality to be composed of various properties and relations of whatever kinds and polyacities that it may and apparently does have.

12. Thomas Aquinas, *Commentary on the Metaphysics of Aristotle*, 2 Vols., trans. John Rowan (Chicago: Henry Regnery Co., 1961).

13. *Moderate Realism*, pp. 51-58.

14. Aristotle's *Metaphysics*, translated by W. D. Ross and found in Richard McKeon, *The Basic Works of Aristotle* (New York: Random House, 1941).

15. Thomas Aquinas, *Summa Theologica*, 3 Vols., trans. Fathers of the English Dominican Province (New York: Benzigner Brothers, Inc., 1947).

16. Thomas Aquinas, *On the Truth of the Catholic Faith (Summa Contra Gentiles), Book One: God*, trans. by Anton Pegis (Garden City, NY: Doubleday & Co., Inc., 1955).

17. John Duns Scotus, *Ordinatio*, II. d.3, part 1, qq. 1-6, in Paul Spade, *Five Texts on the Mediaeval Problem of Universals* (Indianapolis, IN: Hackett Publishing Co., 1994), pp.101.

18. Thomas Aquinas, *On Spiritual Creatures (De Spiritualibus Creaturis)*, trans. M. Fitzpatrick and J. Wellmuth (Milwaukee: Marquette University Press, 1949).

19. Francisco Suarez, *On the Various Kinds of Distinctions (Disputationes Metaphysicae, Disputation VII, de variis distinctionum generibus)*, trans. Cyril Vollert, S.J., (Milwaukee: Marquette University Press, 1947).

20. For arguments against bare particulars see my 'Individuation and Instance Ontology'. For an argument that even Aristotle, from which the tradition of individuating prime matter originated, would have rejected the notion of an absolutely qualityless substrate see Theodore Scaltsas, *Substances and Universals in Aristotle's Metaphysics* (Ithaca, NY: Cornell University Press, 1994), pp. 222-28.

21. This was the view of Keith Campbell in *Abstract Particulars* (Oxford: Basil Blackwell, 1990). Campbell has subsequently modified his views to allow for certain relations that resist foundational reduction. See his 'Unit Properties, Relations, and Spatio-Temporal Naturalism', *The Modern Schoolman* LXXIX (2002), pp. 151-62.

22. See Mark Henninger, *Relations: Medieval Theories*, 1250-1325 (Oxford: Clarendon Press, 1989), and Jeffrey Brower, 'Medieval Theories of Relations', *Stanford Encyclopedia of Philosophy.*

23. William of Ockham, *Ordinatio*, d.2, qq. 4-8, in Paul Spade, *Five Texts on the Mediaeval Problem of Universals*, p. 172.

24. Herbert Hochberg, 'The Wiener-Kuratowski Procedure and the Analysis of Order', *Analysis* 41 (1981), pp. 161-63.

25. *Moderate Realism*, pp. 51-58.

26. Bertrand Russell, *The Principles of Mathematics*, 2d. ed., (1903: reprt. ed., New York: Norton, 1938), pp. 221ff.

27. *Moderate Realism*, pp. 163-73.

28. Bradley, 'Relations', cited in Note 9.

29. Ibid., p. 672.

30. William of Ockham, *Ockham's Theory of Terms: Part I of the Summa Logicae*, trans. by Michael Loux (Notre Dame, IN: University of Notre Dame Press, 1974), p. 170.

31. Bertrand Russell, 'Some Explanations in Reply to Mr. Bradley', *Mind* 19 (1908), pp. 373-8. Also see my 'Individuation and Instance Ontology'.

32. Gustav Bergmann, *Realism* (Madison, WI: University of Wisconsin Press, 1967), pp. 9, 42ff; Herbert Hochberg, 'A Refutation of Moderate Nominalism,' *Australasian Journal of Philosophy* 66 (1988), pp. 188-207; P. F. Strawson, *Individuals* (London: Methuen, 1971), pp. 168ff.

33. See my 'Combinatorial Predication' and 'Individuation and Instance Ontology'.

34. Ibid.

35. Ibid.

36. E.g., Scotus, *Ordinatio* II. d.3, part 1, qq. 1-6, in Spade, *Five Texts on the Mediaeval Problem of Universals*, pp.101-02.

37. D. M. Armstrong, *A World of States of Affairs*, pp. 68, 109; James P. Moreland, 'Theories of Individuation: A Reconsideration of Bare Particulars', *Pacific Philosophical Quarterly* 79 (1998), pp. 251-63.

38. Michael Loux, *Metaphysics: A Contemporary Introduction* (New York: Routledge, 1998), pp. 117ff.; E. J. Lowe, *The Possibility of Metaphysics* (Oxford: Clarendon Press, 1998), pp. 180-83, 197.

39. For Ockham reference, see Note 23; Campbell, *Abstract Particulars*, p.69.

40. See Note 19. For an analogical explanation of the formal distinction see 'Individuation and Instance Ontology'.

41. Campbell, *Abstract Particulars*, p. 56.

42. D. M. Armstrong, *Nominalism & Realism*, pp. 109, 111; and *States of Affairs*, pp. 114-19.

43. The Ontic Predication Thesis in the form of **O″** incorporates two theses given separately in my *Moderate Realism*: the Principles of Immanent Instance Realism (**IR**), p. 11, and Instance Predicates (**IP**), p. 26.

44. **Id** is given in its PPL formalization (utilizing the device of 'extended binding' by intension quantifiers) in 'The Logic of Instance Ontology', and *Moderate Realism*, p. 213.

45. Peter Simons, *Parts: A Study in Ontology* (Oxford: Clarendon Press, 1987), p. 327.

46. One disanalogy of spatial diagrams for depicting *n*-adic relations for $n > 2$ is that it gives the impression that all such relations are reducible to conjunctions of dyadic relations. That this is not possible see my 'Peirce: Logic, Categories, and Triads', *Transactions of the Charles S. Peirce Society* XV (1979), pp. 158-75.

47. Steven French, 'Identity and Individuality in Classical and Quantum Physics', *Australasian Journal of Philosophy* 67 (1989), pp. 432-46.

48. Steven French and Decio Krause, 'Quantum Objects are Vague Objects', *Sorites* 6 (1996), pp. 21-33.

49. French and Ladyman, 'Remodelling Structural Realism'.

50. See Bernard Katz, 'The identity of Indiscernibles Revisited', *Philosophical Studies* 44 (1983), pp. 37-44. Also Richard Swinburne, 'Thisness', *Australasian Journal of Philosophy* 73 (1995), pp. 389-400.

51. This terminology is used in the overview article by Peter Forrest, 'The Identity of Indiscernibles', *The Stanford Encyclopedia of Philosophy* (Summer 2002 Edition), Edward N. Zalta (ed.), URL = <http://plato.stanford.edu/archives/sum2002/entries/ identity-indiscernible/>.

52. Gottfried W. Leibniz, 'The Monadology' in *Goffried Wilhelm Leibniz: Philosophical Papers and Letters*, 2d. ed., trans. & ed. by Leroy Loemker (Dordrecht: Reidel, 1969), p. 467.

53. Loux, *Metaphysics*, p. 107.

54. **IND** corrects the definition of indiscernibility that I had given previously (e.g., in *Moderate Realism* and 'The Logic of Instance Ontology'), viz., what is a liberalization on the conditions of **Id**:

> Entities *a* and *b* are indiscernible if and only if, for every monadic property P^1, there is an instance P^1_i of P^1 such that $P^1_i(a)$ if and only if there is an instance P^1_j of P^1 such that $P^1_j(b)$.

The idea is that entities having as predicates instances all and only of the same properties are indiscernible, and if, for one or more properties, each entity has an instance of it non-identical to that of the other entity, then the entities will be indiscernible but not identical. The problem here, as with the traditional definition of indiscernibility, is finding a principled way to exclude from the range of P^1 trivializing properties like 'is identical to *a*'. **IND** avoids all this by having indiscernibility turn upon the internal constituents composing entities and making them to be what the are, and not what can be externally predicated of them and presupposing them.